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SOME METHODS OF SOLVING HIGHER DEGREE EQUATIONS

[1]Nurbayev A.R, [2]Xolboyeva Z

[1] Teacher, Gulistan State University

[2]Student, Gulistan State University

Annotation

This research is conducted to describe some methods of solving higher degree equations and their differences from other methods. New recommendations for solving higher degree equations are given. Mainly quadratic and cubic equations are considered. Special substitutions are used to solve the cubic equation.

The section on equations is one of the main parts of learning mathematics. Algebraic equations are partially taught in secondary and higher educational institutions. There is a method of solving high-degree algebraic equations up to the fourth-degree equation. That is, the roots of the equation can be expressed through the coefficients of the equation. There is no such method for algebraic equations of the fifth degree and higher. But there are some attempts and simplifications. For example, using substitutions, some coefficients of nth degree equation can be brought forward to zero. Let's take a look at some of them.

It is sufficient to use the simple case of Chirengauz substitution in the quadratic equation. Let us be given the following quadratic equation:

$$ax^2+bx+c=0 \quad (a\neq 0)$$

We express it as follows:

$$x^2+px+q=0 \quad (p=b/a, q=c/a)$$

We do substitution as follows:

$$x=y+t$$

We determine t by turning the coefficient of the y to zero.

$$(y+t)^2 + p(y+t) + q = 0$$
$$y^2 + (2t+p)y + t^2 + pt + q = 0$$
$$2t + p = 0$$
$$t = -\frac{p}{2}$$

So the value of t is determined. After replacing t the coefficient of the 1st degree unknown number disappears. The equation becomes simpler:

$$y^2 + r = 0$$

from here the value of r is determined.

The cubic equation was solved geometrically for the first time by Umar Khayyam (1048-1123) in the 11th century. He divided the third-degree equation into circular and parabolic equations and then he proved the point of their intersection is the solution of the given equation. In this proof, the coordinate system was directed from left to right and from top to bottom.

Italian Ferro (1465-1526) at the beginning of the 16th century found the method to solve the following equation:

$$x^3 + px + q = 0$$

We show a method that is significantly different from the one mentioned above. We recommend the general solution of the equation:

$$x^3 + ax^2 + bx + c = 0 \qquad (1)$$

Firstly, we do the substitution for (1):

$$x = y - \frac{a}{3}$$

In this case, we have the equation:

$$y^3 + py + q = 0 \qquad (2)$$

Then, we can obtain the equation $z^3 + z + r = 0$ (3) by doing this substitution: $y = \sqrt{p}z$.

(3) is turned into

$$t^3 - \frac{1}{27t^3} + r = 0 \quad (4) \text{ by doing this substitution: } z = t - \frac{1}{3t}$$

By doing this $t^3 = u$ substitution (4) becomes the following quadratic equation:

$$27u^2 + 27ru - 1 = 0$$

If we find the solutions u_1, u_2 from this quadratic equation and put it in $t^3 = u$, t_{11}, t_{12}, t_{13} and t_{21}, t_{22}, t_{23} are derived. Using these z_{11}, z_{12}, z_{13} and z_{21}, z_{22}, z_{23} are found. y is found with the help of z's, and then x the solution of the equation is found.

Example

$$27x^3 + 81x^2 + 108x + 28 = 0$$

Solve the equation.

Solution. We write the equation in this form:

$$x^3 + 3x^2 + 4x + \frac{28}{27} = 0$$

By doing this $x = y - 1$ substitution the equation becomes

$$y^3 + y - \frac{26}{27} = 0$$

We obtain $z^3 - \frac{1}{27z^3} - \frac{26}{27} = 0$ by the substitution: $y = z - \frac{1}{3z}$

By doing this $z^3 = u$ substitution the equation becomes the following quadratic equation:

$$27u^2 - 26u - 1 = 0$$

We solve it : $u_1 = 1$ and $u_2 = -\frac{1}{27}$

Then, z and y are found, and then x's the roots of equation are found:

$x_1 = -\frac{1}{3}$, $x_2 = \frac{-4+2\sqrt{3}i}{3}$, $x_3 = \frac{-4-2\sqrt{3}i}{3}$

bo`ladi. These solutions completely satisfy the equation.

Overall, by doing some substitutions we can change the algebraic equation

$x^n + a_1 x^{n-1} + ... + a_{n-1}x + a_n = 0$ to this

$y^n + b_1 y^{n-4} + ... + b_{n-1}y + b_n = 0$

This method is very useful when solving some high-degree equations. If we do some substitutions to solve the fifth-degree equation:

$x^5 + ax^4 + bx^3 + cx^2 + dx + e = 0$ it becomes simpler

$y^5 + py + q = 0$

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