PHRAGMEN-LINDELOF TYPE THEOREM AND FUNCTIONS

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Annotation

In this article we consider some estimates of the Carleman's function constructed by U.Yu.Juraeva, to find integral representation for the biharmonic functions($\Delta^2 u(y) = 0$)defined in unbounded domain $D = \left\{y: y = (y_1, y_2), -\infty < y_1 < \infty, 0 < y_2 < \frac{\pi}{\rho}, \rho > 0\right\}$ of two dimensional Euclidean space.

Key words. Phragmen-Lindelof type theorem, Carleman's functions, byharmonic functions, integral representation.

We consider the problem: Let u(P) be a harmonic function solved in an infinite domain D in a two-dimensional Euclidean space, continuous up to the boundary of the domain with its first-order derivatives. It is required to show that if a function and its normal derivative are bounded on the boundary D and u(P) is unbounded inside, then for P→∞ it must grow inside D at a rate not less than some limit, and estimate this limit growth rate. In the articles E.M.Landis[1],M.A.Evgrafov, I.A.Chegis[2], A.F.Leontiev[4], I.S.Arshon[3], Leontiev A.F.[4], Armstrong-Sirakov-Smart [5], AdamowiczT[6],JinZ., Lancaster K [7], Sh. Yarmukhamedov,[8]-[9], Ashurova Zebiniso Rakhimovna [12], N. Zhuraeva[14], At Zhuraev[17], the possible rates of increase and decrease of a solution defined in an unbounded domain when a point is removed to infinity are established. This rate of increase or decrease depends on the shape of the region and the nature of the boundary conditions. The possible rates of increase and decrease of the solution in the vicinity of the boundary point are also clarified, depending on the structure of the boundary in the vicinity of this point and on the nature of the boundary conditions. For example: A. F. Leontiev in the article- On Fragment–Lindelof type theorems for harmonic functions in a cylinder, Izv. AN USSR. Ser. math., 1963, volume 27, issue 3, 661-676. received the following theorem.

Theorem 2. Let u(r,ϕ,x) be a harmonic function. in a round cylinder :$0 \leq r \leq a$, $0 \leq \phi < 2\pi$, $-\infty < x < \infty$. If the condition is met

$$u(a,\phi,x)=0, \left|\frac{\partial u}{\partial r}(r,\phi,x))\right| < cexp\mu(x),$$

$$\max_{(r,\phi)} |u(r,\phi,x)| < c\, exp\, exp \frac{\pi|x|}{2(a+\varepsilon)}, \varepsilon > 0$$

$$\text{тогда} \quad u(r,\phi,x) \equiv 0$$

with $\mu(x) < \frac{\propto}{a}$, where ∝-the smallest positive zero of the Bessel function $J_0(x)$, $u(r,\phi,x) \equiv 0$.

Using the ideas of M.M.Lavrentiev, Sh. Yarmukhamedov in his works for the first time offers a method for constructing a family of fundamental solutions of the Laplace equation. He obtained Green's integral formula in an unbounded domain in the class of growing harmonic functions. In this direction, he established a Fragment-Lindelof type theorem for harmonic functions.

Zebiniso Rakhimovna Ashurova obtained several Fragment-Lindelof-type theorems for harmonic functions of many variables in [12].

In 2009, Zhuraeva N. obtained regularization and solvability of the Cauchy problem for polyharmonic equations of order n in some unbounded domains (for arbitrary odd m and even m when 2n<m) [4]-[8]. Later, together with Zhuraeva, in 2009, he solved this problem for some unbounded domains (for arbitrary even m when 2n≥m). Ashurova Zebiniso Rakhimovna, Nodira Juraeva and Umida Juraeva together for harmonic functions of two variables[13]-[15].

In this paper, some estimates of the growth of the Carleman function constructed by U.Zhuraeva inside the regions of the strip type are given.

Пусть R^2 - двухмерное вещественное евклидово пространство,

$$x = (x_1, x_2), x' = (x_1, 0), \ r = |x - y|, \ s = |x' - y'|, \ \alpha^2 = s,$$

$$D = \left\{y: y = (y_1, y_2), y_1 \in R, 0 < y_2 < h, h = \frac{\pi}{\rho}, \ \rho > 0\right\}.$$

Функцию $\Phi_\sigma(y,x)$при $s > 0, \sigma \geq 0, a \geq 0$, определим:

$$\Phi_\sigma(y,x) = \boldsymbol{c_0}(8\pi)^{-1} \int_{\sqrt{s}}^{\infty} \mathrm{Im}\left[\frac{\exp(\sigma\omega+\omega^2) - achi\rho_1\left(\omega-\frac{h}{2}\right)}{\omega - x_2}\right](u^2 - s)du \,, \omega = iu + y_2, \qquad (3)$$

Theorem 1. $\Phi_\sigma(y,x)$ is a polyharmonic function of order 2 in y when s> 0 and for this function there is a

$$\Phi_\sigma(y,x) = c_0\left(r^2 \ln r + G_\sigma(x,y)\right),$$

Where $G_\sigma(y,x)$ regular in variable y and continuously differentiable by $\overline{D}$.

Theorem 2. For the function $\Phi_\sigma(y,x)$which can be represented as$\Phi_\sigma(y,x) = r^2(J_1^1 - J_1^2)$ takes place

$$\left|\frac{\partial J_1^1}{\partial y_1}\right| \le \left(\frac{1}{r^2}+\frac{1}{r^3}\right)\frac{c_0}{exp(A)}, \qquad \left|\frac{\partial J_1^2}{\partial y_1}\right| \le \left(\frac{1}{r^3}\right)\frac{c_0}{exp(A)}$$

$$\left|\frac{\partial J_1^1}{\partial y_2}\right| \le \left(1+\frac{1}{r}+\frac{1}{r^2}\right)\frac{c_0}{exp(A)}, \quad \frac{\partial J_1^2}{\partial y_2} \le \left(\frac{1}{r^2}+\frac{1}{r^3}\right)\frac{c_0}{exp(A)}$$

where, $A = ach\alpha\rho_1$,$Q = exp(\sigma y_2 + {y_2}^2)$,

$$A_2 = \left((\sigma+2y_2)\sqrt{r^2t+s} \pm asin\rho_1\left(y_2-\frac{h}{2}\right)sh\rho_1\sqrt{r^2t+s}\right),$$

$$J_1^1 = \int_0^\infty \frac{Q(y_2-x_2)sinA_2}{exp\left(ach\rho_1\sqrt{r^2t+s}cos\rho_1\left(y_2-\frac{h}{2}\right)\right)exp(r^2t+s)(t+1)}\frac{tdt}{\sqrt{r^2t+s}}$$

$$J_1^2 = \int_0^\infty \frac{Qcos A_2}{exp\left(ach\rho_1\sqrt{r^2t+s}cos\rho_1\left(y_2-\frac{h}{2}\right)\right)(t+1)}\frac{tdt}{exp(r^2t+s)}$$

Theorem 2. Let the external normal to the boundary be ∂D. Then the inequality is valid for the function Φ(y,x):

$$|\Phi_\sigma(y,x)| \le \left(\frac{\sigma}{r}+\frac{1}{r^2}\right)\frac{r^2 c_0}{exp(A)} = \frac{c_0(\sigma r+1)}{exp(A)}.$$

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