

BARYCENTRIC COORDINATES AND THEIR APPLICATIONS PHYSICAL AND GEOMETRIC MEANING.

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Abstract. This article discusses the process of increasing the independent learning opportunities of future mathematics and physics teachers in digital learning environments. barycentric coordinates and their applications physical and geometric meaning The content of using digital technologies is highlighted, and the level of effectiveness of the results obtained based on experiments is determined.

Keywords: barycentric, point, barycentric coordinates, talent, motivation, innovation, creativity, critical thinking, problem solving. student work.

Introduction. Myobius is considered the founder of barycentric calculus, a branch of analytic geometry that studies algebraic operations on points in affine or Euclidean space. Barycentric calculus was not very developed in the 19th century, but later it, and especially the method of barycentric coordinates introduced by Myobius, found various applications.

Barycentric coordinates are scalar parameters whose set defines a unique point in an affine space. Let P be an arbitrary point in A . Every point $M \in A$ can be uniquely represented as a barycentric representation.

$$M = P + \alpha_0 \overrightarrow{PP_0} + \alpha_1 \overrightarrow{PP_1} + \dots + \alpha_n \overrightarrow{PP_n};$$

Points in the linear combination on the right $\alpha_0, \alpha_1, \dots, \alpha_n$ satisfies the following condition.

$$\alpha_0 + \alpha_1 + \dots + \alpha_n = 1$$

$\alpha_0, \alpha_1, \dots, \alpha_n$ These are called the barycentric coordinates of point M.

The above equation can be rewritten as:

$$M = \alpha_0 P_0 + \alpha_1 P_1 + \dots + \alpha_n P_n$$

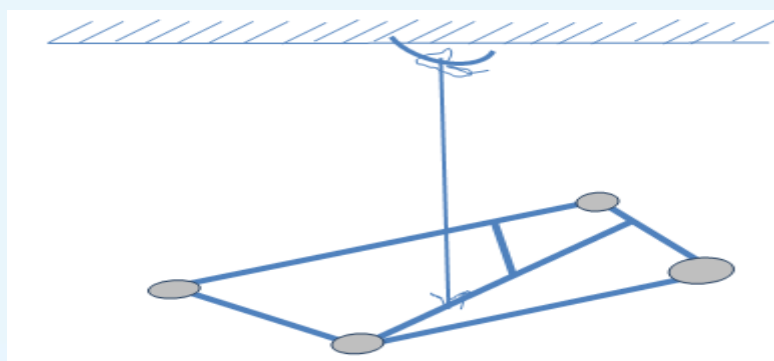
Barycentric coordinates. Let us be given a triangle. Then any point in the plane of the triangle can be described as the center of some masses located at its vertices. The set of numbers is called the barycentric coordinates of the point with respect to the triangle. ABC P a, b, c (a, b, c)

The barycentric coordinates of a point are multiplied by a nonzero factor is defined as any three numbers that all define the same point. Any three numbers whose sum is not zero are the barycentric coordinates of a point. Barycentric coordinates are sometimes called proportional triple coordinates whose sum is one. $k \neq 0$ (ka, kb, kc)

If a point is located inside a triangle, its barycentric coordinates are proportional to the areas of the triangles. P ABC PAB, PBC, PCA

Mechanical and geometric meaning of the concept of barycenter

If we take objects with masses, their position in space as material points consists of a system. If we connect the system of material points in a general way, that is, if we place them on a rod and hang the rod from an arbitrary point to a thread, this system will be in unstable equilibrium or the rod system will deviate. However, there is such a wonderful point in the system that the material points remain in a stationary state. $m_1, m_2, \dots, m_n$ $A_1, A_2, \dots, A_n$



Drawing 1.

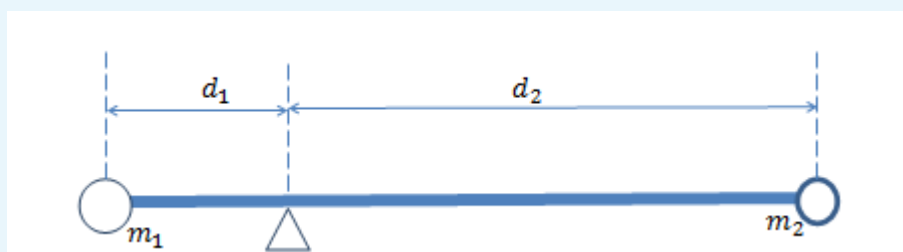
We take four material points as a system and combine them into one system.

If we take an arbitrary point from the combined rod and hang the entire system from this point to a string, part of the system, that is, the rod, will deviate, but there is such a point in the system that if we hang the system from this point to a vertical string and release it at an arbitrary angle around the point of the system, it will be in equilibrium or at rest. This point is called the center of gravity or barycenter of the system of material points. $Z \quad Z - Z -$

The following simple and mechanical properties of the concept of center of gravity are used in solving geometric problems.

1. Any system consisting of a finite number of material points has a center of gravity and is unique.

2. The center of gravity of two material points is defined according to Archimedes' principle, which is located at the intersection connecting these points (according to the golden rule of mechanics, we place the material points on the shoulders) $d_1, d_2 - m_1, m_2 -$.



Drawing 2.

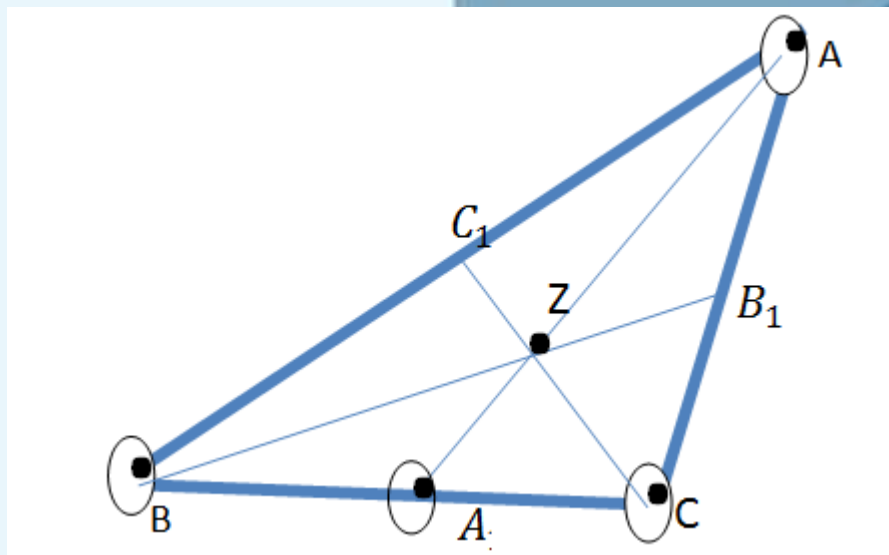
3. If we assign masses of a certain value to each of a finite number of material points and replace the masses of several of them, the center of gravity of this system will not change.

Theories seem to state that these facts are presented in a very simple framework in mechanics. But we prove these properties through formulas.

Archimedes' theorem.

The three medians of a triangle intersect at a single point and divide the triangle at the point of intersection in a ratio of 2:1, counting from the vertex.

Proof. Let , , be the medians of the triangle. $ABC \quad AA_1 \quad BB_1 \quad CC_1$



Drawing 3.

A, B, C – We hang a load of 1 gram on the ends. The resulting system of three material points has a center of gravity. According to property 3, if we move the mass of the material points to the point where their center of gravity is, the position of the center of gravity does not change, but the Z-point and the center of gravity of the material points remain the same. A_1, B_1, C_1 So, just like that $Z \in [AA_1]$
 $Z \in [BB_1]$ and $Z \in [CC_1]$

Thus, the center of gravity of the three material points is the Z-point. According to the lever rule, the following equality holds:

$$2|ZA_1| = 1|ZA| \text{ and } |ZA| : |ZA_1| = 2 : 1.$$

Define the concept of center of gravity in mathematics.

Determining the concept of the center of gravity by hanging from the center of gravity on a string is inconvenient for finding an exact mathematical solution to geometric problems. Although this physical appearance is constantly in our imagination, we must determine the mathematical content of the concept of the center of gravity using geometric terms. The expression material point represents the mass m placed on the point. $m_A - A$

To determine the definition of the center of gravity in mathematics, we first take a heuristic look. We consider two material points, for which the point is the center of gravity (the lever rule), we can write the following equations. $m_1 A_1 m_2 A_2 Z -$
1. , $m_1 |\overrightarrow{ZA_1}| = m_2 |\overrightarrow{ZA_2}|$

$$2. \cdot |m_1 \overrightarrow{ZA_1}| = |m_2 \overrightarrow{ZA_2}|$$

$\overrightarrow{ZA_1}$ and the opposite vectors from this $\overrightarrow{ZA_2}$

$$m_1 \overrightarrow{ZA_1} = -m_2 \overrightarrow{ZA_2}, \quad m_1 \overrightarrow{ZA_1} + m_2 \overrightarrow{ZA_2} = \vec{0}$$

we will have equality

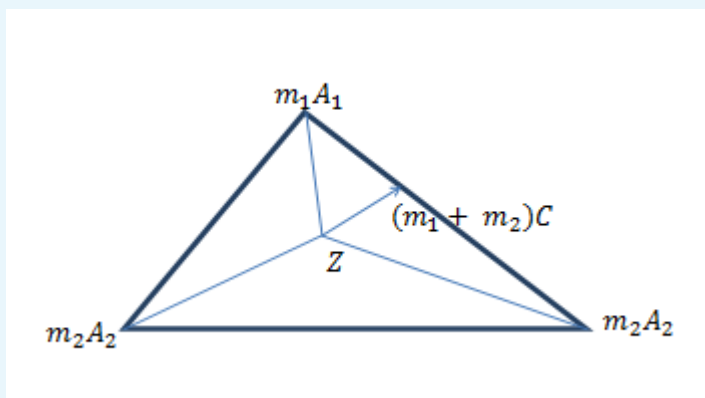
It can be seen that the point must be made to be the center of gravity of the material points. $Z - m_1 A_1 m_2 A_2$ $m_1 \overrightarrow{ZA_1} + m_2 \overrightarrow{ZA_2} = \vec{0}$

$m_1 A_1$, the center of gravity of material points $m_2 A_2, m_3 A_3$

$Z - m_1 A_1$, while those of material points are points $m_2 A_2 - C$

$m_1 \overrightarrow{CA_1} + m_2 \overrightarrow{CA_2} = \vec{0}$ The equality is fulfilled. The center of gravity of the three material points is at the center of gravity of (and), that is, at the point. $m_1 + m_2)C - m_3 A_3 Z -$

$$(m_1 + m_2) \overrightarrow{ZC} = (m_1 \overrightarrow{ZC} + m_2 \overrightarrow{ZC}) = m_1 (\overrightarrow{ZA_1} - \overrightarrow{CA_1}) + m_2 (\overrightarrow{ZA_2} - \overrightarrow{CA_2}) = m_1 \overrightarrow{ZA_1} + m_2 \overrightarrow{ZA_2} - (m_1 \overrightarrow{CA_1} + m_2 \overrightarrow{CA_2}) = m_1 \overrightarrow{ZA_1} + m_2 \overrightarrow{ZA_2}$$



Drawing 4.

It can be seen from this that the center of gravity or barycenter of a system of material points is such a point for which the following equality must be satisfied: $m_1 A_1 m_2 A_2, \dots, m_n A_n Z$

$$m_1 \overrightarrow{ZA_1} + m_2 \overrightarrow{ZA_2} + \dots + m_n \overrightarrow{ZA_n} = \vec{0}.$$

According to the arguments presented, it is impossible to prove this equality. However, we can say that the masses are placed in correspondence with the points. $m_1, m_2, \dots, m_n A_1, A_2, \dots, A_n$

The barycenter is called the centroid for polygons and polyhedra.

Theorem 1 If the center of gravity of material points is a point, then the following equality holds for an arbitrary point in space: $m_1 A_1 m_2 A_2, \dots, m_n A_n ZO$

$$\overrightarrow{OZ} = \frac{m_1 \overrightarrow{OA_1} + m_2 \overrightarrow{OA_2} + \dots + m_n \overrightarrow{OA_n}}{m_1 + m_2 + \dots + m_n} \text{ and vice versa, if the above equality}$$

holds for an arbitrary point in space, the point will be the center of gravity of the system. $OZ m_1 A_1 m_2 A_2, \dots, m_n A_n$

Proof. Support for two material points

$$m_1 \overrightarrow{ZA_1} + m_2 \overrightarrow{ZA_2} = \vec{0} \text{ We find by changing the equation from}$$

$$. m_1 (\overrightarrow{OA_1} - \overrightarrow{OZ}) + m_2 (\overrightarrow{OA_2} - \overrightarrow{OZ}) = \vec{0} \overrightarrow{OZ}$$

$$\overrightarrow{OZ} = \frac{m_1 \overrightarrow{OA_1} + m_2 \overrightarrow{OA_2}}{m_1 + m_2} .$$

Theorem 2. The center of gravity of two material points is at the intersection where they lie and is determined by Archimedes' principle: $m_1 d_1 = m_2 d_2$.

Proof., It can be seen from the equalities that the vectors and lie on the cross-section in opposite directions. The principle of equal strength Archimedes' lever is valid. $m_1 \overrightarrow{ZA_1} + m_2 \overrightarrow{ZA_2} = \vec{0} m_1 \overrightarrow{ZA_1} = - m_2 \overrightarrow{ZA_2} \overrightarrow{ZA_1} \overrightarrow{ZA_2} A_1 A_2 m_1 |\overrightarrow{ZA_1}| = m_2 |\overrightarrow{ZA_2}| , m_1 d_1 = m_2 d_2$

Conclusion

In conclusion, it can be said that barycentric coordinates can be used to find the sides of polygons, the relationship between them, the ratio of sections, the areas, and the volumes.

Through this, we can see the connection between mathematics and physics and the enrichment of the content of elementary problems.

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