

TRIGONOMETRIC FUNCTIONS IN THE GALILEAN AND MINKOWSKI PLANES.

Abdurashid Nurbayev. Gulistan State University, teacher

Jurabek Abdullayev. Dehqanabad Vocational School, teacher

Abstract

This article analyzes the properties of trigonometric functions in the Galilean and Minkowski planes. In Galilean geometry, space and time are independent, and the classical trigonometric functions ($\sin x$, $\cos x$, $\tan x$, $\cot x$) are used in a slightly different way. In Minkowski geometry, the space-time interval is defined based on the theory of relativity, and hyperbolic trigonometric functions ($\sinh x$, $\cosh x$, $\tanh x$, $\coth x$) are used. This differences space-time concepts physicist basics and their mathematician to be expressed deeper to understand help gives .

Key words : Galilean geometry , Minkowski plane , sinus, cosinus , trigonometric function , circle , parabola, hyperbola.

Trigonometry on the Galilean plane

In non-Euclidean geometry, each geometry has its own trigonometric functions. We will consider trigonometric functions in the Galilean plane and the Minkowski plane. For this, we need to know the dimensions and concepts of these planes well. We can learn about them in [1],[2].

We G_2 find trigonometric functions in the (Galilean plane) using the theory of general trigonometry.

G_2 A parabola is taken as a circle in space . We choose the parabola so that the axis of symmetry OY passes through the points (1;0) and (0;1).

So , the vertex of the parabola is we determine a $x_0 = 0$, $y_0 = 1$, c from the general equation $y = a + bx + cx^2$

$$x_0 = 0, b=0$$

$$(1;0) \Rightarrow a+c=0, y_0 = 1, \frac{4ac-b^2}{4a} = 1 \quad c=1 \quad a=-1$$

$y = -x^2 + 1$ G_2 is taken as a unit circle in space.

$$y + x^2 = 1 \quad (1)$$

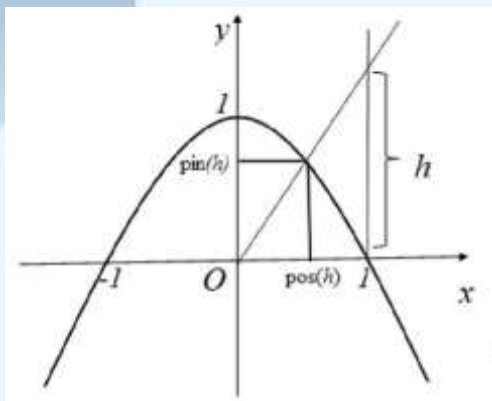


Figure 1

G_2 Let's first consider the angle (h) in .

h in the corner unity our circle coordinates above to the picture according to is determined .

Definition 1: h corner $pin(h)$ is the point (1; 0) h to the corner in a hurry harvest was of the point coordinates head with connector correct line , and $x^2 + y = 1$ around , from the intersection harvest was of the point to the ordinate It is said .

Definition 2: h corner $pos(h)$ is the point (1; 0) h to the corner in a hurry harvest was of the point coordinates head with connector correct line , and $x^2 + y = 1$ around , from the intersection harvest was of the point to the abscissa It is said .

To be selected according to pin and for posh following fact appropriate :

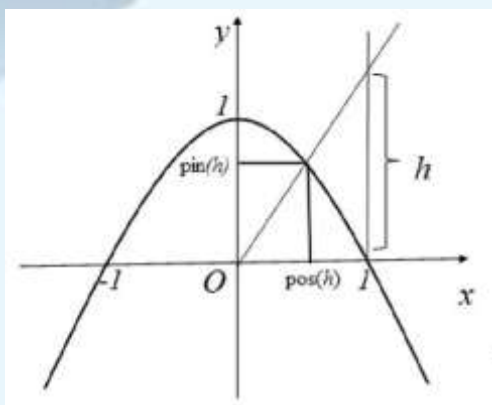
$$pos^2 h + pin h = 1 \quad (2)$$

Proof : $h = h_0$ at $x = pos h_0$ $y = pin h_0$

x_0, y_0 point because $x^2 + y = 1$ it belongs to $x_0^2 + y_0 = 1$

$pos^2 h + pin h = 1$ comes from.

By definition, let's look at the value of $pin h_0 = h_0$,



$$x_0 = x, y_0 = y, h_0 = h$$

$$\begin{cases} x^2 + y - 1 = 0 \\ y = hx \end{cases} \quad (3)$$

Figure 2

The solution to system (3) is $x \rightarrow \text{posh}$ ni value, $y \rightarrow \text{pinh}$ ni value.

$$x^2 + hx - 1 = 0 \Rightarrow x = \frac{-h + \sqrt{h^2 + 4}}{2} \Rightarrow \text{posh} = \frac{-h + \sqrt{h^2 + 4}}{2}$$

$$y = \frac{h(-h + \sqrt{h^2 + 4})}{2} = \frac{2h}{\sqrt{h^2 + 4} + h} \text{pinh} = \frac{2h}{\sqrt{h^2 + 4} + h}$$

$$\begin{cases} \text{posh} = \frac{-h + \sqrt{h^2 + 4}}{2} \\ \text{pinh} = \frac{2h}{\sqrt{h^2 + 4} + h} \end{cases} \quad (4)$$

$$\begin{cases} \text{ptgh} = \frac{\text{pinh}}{\text{posh}} = h \\ \text{ctgh} = \frac{\text{posh}}{\text{pinh}} = \frac{1}{h} \end{cases} \quad (5)$$

According to (4) at $h=0$, $\text{pos}0=1$, $\text{pin}0=0$, $\text{pos}(00)=0$, $\text{pin}(00)=1$

$\text{pin} < 1$

If the angle h is determined, (ah) can also be determined using the same method as $\text{pin}(ah)$, $\text{pos}(ah)$.

$$x^2 + y - 1 = 0$$

$$y=ahx$$

$$\begin{cases} \text{pinh} = \frac{2ah}{\sqrt{a^2h^2 + 4} + ah} \\ \text{posh} = \frac{-ah + \sqrt{a^2h^2 + 4}}{2} \end{cases}$$

From (4) and (5), the following facts can be proven.

1. $\text{posh} - \text{pos}(-h) = -h$
2. $\text{pinh} - \text{pin}(-h) = -h^2$
3. $\text{pinh} \times \text{pos}(-h) = h$
4. $\text{ptgh} \times \text{pctgh} = 1$

Galileo on the plain trigonometric functions one how many properties brought let's go :

- $\text{pin}(h)$ and $\text{pos}(h)$ functions periodic it's not
- $\text{pin}(h)$ and $\text{pos}(h)$ functions not even odd not even

Now the posh function periodic that it is not we will show .

$$\text{pos}(h+T) = \frac{-h-T+\sqrt{(h+T)^2+4}}{2} = \frac{-h+\sqrt{h^2+4}}{2}$$

$$-T + \sqrt{(h+T)^2 + 4} = \sqrt{h^2 + 4}$$

$$h^2 + 2hT + T^2 + 4 = h^2 + 4 + T^2 + 2T\sqrt{h^2 + 4}$$

$$hT = T\sqrt{h^2 + 4}$$

$$T(h - \sqrt{h^2 + 4}) = 0$$

So , posh is periodic. function it's not .

Minkowski on the plain trigonometry

It is known that the traditional trigonometric functions of sine and cosine express the main difference between a right triangle and a right angled triangle through the basic trigonometric identity:

$$\sin^2(\alpha) + \cos^2(\alpha) = 1$$

The Pythagorean theorem, as well as the basic formulas of the basic trigonometric functions, are expressed by $\cos\alpha = \frac{a}{c}$ and $\sin\alpha = \frac{b}{c}$.

Now we will consider the main properties that connect the hyperbolic functions of cosine and sine:

$$\text{Ch}^2 \alpha - \text{Sh}^2 \alpha = 1 \quad (6)$$

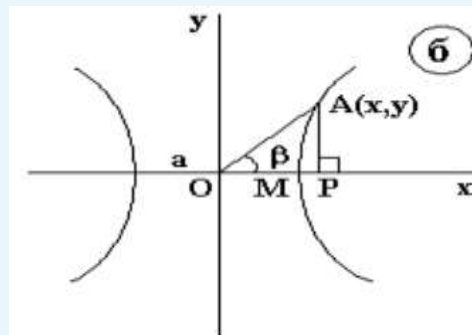


Figure 3

$x^2 - y^2 = 1$ We choose a point $A(x, y)$ from the equation of the hyperbola. And we connect the point A with the coordinate origin $(O(0, 0))$. β We denote the angle between OA and the OX axis as β . We choose the projection of point A on the OX axis as hyperbolic sine, and the projection on the OY axis as hyperbolic cosine. Therefore, our point $A(x, y)$ takes the form $A(\text{cht}, \text{sht})$.

At $t=0$, the arc length is 0.

$$(\text{ch}0, \text{sh}0) = (1, 0)$$

$$\text{ch}0 = 1, \text{sh}0 = 0$$

$x^2 - y^2 = 1$ According to the equation and the formula for calculating the arc length,

$$\int_a^b \sqrt{1 + (f'(x))^2} dx$$

We derive the hyperbolic sine and hyperbolic cosine functions;

$$x^2 - y^2 = 1$$

$x = 1 + y^2$ If we put this equation into the formula for calculating the arc length, we get:

$$\int_0^y \sqrt{1 + \frac{y^2}{1+y^2}} dy = y(t)$$

The integral is formed.

Using the above equations,

$$y(t) = \frac{e^t - e^{-t}}{2}$$

we get the equality. Since $y(t) = \text{sht}$,

$$\text{sht} = \frac{e^t - e^{-t}}{2} \quad (7)$$

It follows that is equal to .

Using these, our hyperbolic sine function is defined as follows:

$$\text{cht} = \frac{e^t + e^{-t}}{2} \quad (8)$$

Functions (7) and (8) are called the hyperbolic sine and hyperbolic cosine functions in the Minkowski plane.

Each plane has the following basic characteristics.

$\sin^2 \alpha + \cos^2 \alpha = 1$ On the Euclidean plane.

$ch^2 x - sh^2 x = 1$ On the Minkowski Plain

Parabolic trigonometry on the plane was introduced by G. Dattoli, M. Migliorati, M. Quattromini, and P. E. Ricci, in which the arc length of the parabola was taken as the angular measure[3].

$$\cos p^2(\varphi) + \sin p(\varphi) = 1$$

Here

$$\begin{aligned} \cos p(\varphi) &= \sqrt[3]{\frac{4-3\varphi}{2} - \sqrt{1 + \frac{(4-3\varphi)^2}{4}}} - \sqrt[3]{\sqrt{1 + \frac{(4-3\varphi)^2}{4}} - \frac{4-3\varphi}{2}} \\ \sin p(\varphi) &= -\sqrt[3]{\left[\frac{4-3\varphi}{2} - \sqrt{1 + \frac{(4-3\varphi)^2}{4}}\right]^2} - \sqrt[3]{\left[\sqrt{1 + \frac{(4-3\varphi)^2}{4}} - \frac{4-3\varphi}{2}\right]^2} + 3 \end{aligned}$$

PCB satisfactory functions . Now Galileo on the plain trigonometric functions we enter . Galileo on the plain The circle is a parabola. for [3], parabolic trigonometry used . In this plane trigonometry Galileo in the sense of corner in size we give .

Description : h corner $pin(h)$ is the point (1; 0) h to the corner in a hurry harvest was of the point coordinates head with connector correct line , and $x^2 + y = 1$ around , from the intersection harvest was of the point to the ordinate It is said .

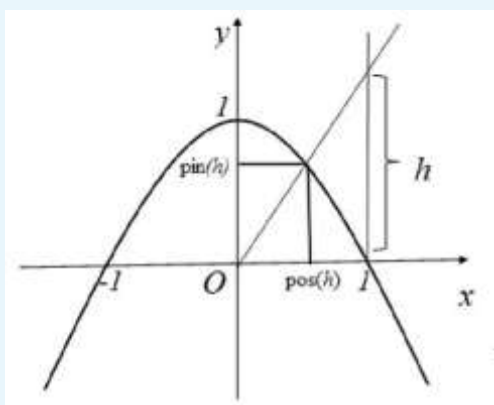
Description : h corner pos (h) is the point (1; 0) h to the corner in a hurry harvest was of the point coordinates head with connector correct line , and $x^2 + y = 1$ around , from the intersection harvest was of the point to the abscissa It is said .

$$\text{pos} (0) = 1 \text{ pin}(0) = 0$$

General in trigonometry designated $C(\varphi | p, q)$ va $S(\varphi | p, q)$ of below to the state correct comes , on the plain Let's see .

$$S(\varphi | 2, 1) = \text{pin}(\varphi) \quad C(\varphi | 2, 1) = \text{pos}(\varphi)$$

but this equalities now Galileo on the plain what we saw for general in trigonometry properties and differences this in trigonometry appropriate It won't be possible .



$$x^2 + y = 1$$

From him any point $M(x, y)$ We get . This point is on the axis Ox projection pos (h), and on the Moon's axis projection to pin(h) we assume equal .

$$\text{pos} (h) = x \text{ pin}(h) = y$$

$$\text{So : } \text{pos}^2(h) + \text{pin}(h) = 1 \quad \text{fact appropriate}$$

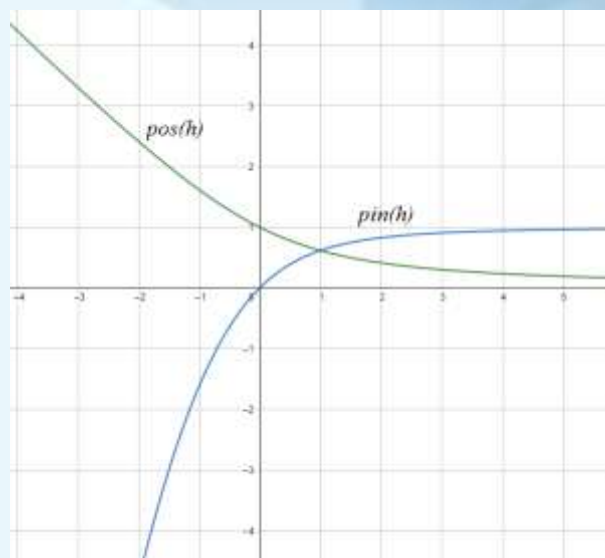
will be .

$$\text{pin}(\infty) = 1 \quad \text{pos}(\infty) = 0$$

$$\text{pos}(h) = \frac{2}{\sqrt{h^2 + 4} + h} \quad \text{pin}(h) = \frac{2h}{\sqrt{h^2 + 4} + h}$$

$$\text{ptg}(h) = \frac{\text{pin}(h)}{\text{pos}(h)} = h \quad \text{ptg}(h) \cdot \text{cth}(h) = 1$$

This is trigonometry. Euclid on the plain included in trigonometry For $\forall m \in \mathbb{R}$, $\sin(m \cdot a)$ and $\cos(m \cdot a)$ by $\sin a$ and $\cos a$ a determination for reveal formula not found, but Galileo flatness in trigonometry $\sin(\alpha h)$ and $\cos(\alpha h)$ determination possible, and as follows :



$$\cos(\alpha \cdot h) = \frac{-\alpha \cdot \sin(h) + \sqrt{\alpha^2 \cdot \sin^2(h) + 4 \cdot \cos^2(h)}}{2} \quad \sin(\alpha \cdot h) = \alpha \cdot h \cdot \cos(\alpha \cdot h)$$

Galileo on the plain trigonometric functions one how many properties brought let's go :

- $\sin(h)$ and $\cos(h)$ functions periodic it's not
- $\sin(h)$ and $\cos(h)$ functions not even odd not even

Theorem : Galileo on the plain $\sin(h)$ and $\cos(h)$ trigonometric functions for below equalities appropriate will be :

$$\begin{cases} \sin(h) + \sin(-h) = -h^2 \\ \cos(h) - \cos(-h) = -h \\ \cos(h) \cdot \cos(-h) = 1 \\ \sin(h) \cdot \sin(-h) = -h^2 \end{cases}$$

GENERAL CONCLUSIONS

The trigonometry department is one of the main departments of mathematics. The role of trigonometric functions in the study of geometry is important. Trigonometric functions have been studied sufficiently in Euclidean space. In the graduation qualification work, the introduction of trigonometric functions, their properties, and angular values were determined. Several issues related to it were considered. In the second part of the work, the introduction of trigonometric functions in non-Euclidean spaces, namely Galilean and Minkowski spaces, the introduction of trigonometric functions, their properties, and angular values were determined. Also, in the Galilean plane, the angle was chosen as an arc of a parabola, and in the Minkowski plane, the angle was chosen as an arc of a hyperbola. It was seen that some properties in Euclidean and non-Euclidean geometry are preserved and some are not. It should be noted that some problems that are worked out in trigonometry in the Euclidean plane can be solved more easily in the non-Euclidean plane.

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