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ON A PURSUIT-EVASION DIFFERENTIAL GAME WITH FOUR PURSUERS AND ONE EVADER

Abstract: *the paper constructs a mathematical model, an effective strategy, and an algorithm in a differential game with four pursuers and one evader. The evader must remain inside the convex hull to ensure capture by the pursuers. The pursuers must adhere to the P-strategy. Afterwards, the improved strategy for the pursuers and the implementation of the algorithm in Python are considered.*

Keywords: *differential game, simple motion, convex hull, P-strategy, improved strategy*

INTRODUCTION

The pursuit-evasion differential games which were originated by Rufus Isaacs in 1965 are special type of mathematical problems closely related to the game and control theory. Main concern of the field is to analyze conflicts that may potentially arise between players in the dynamical systems setting where players have opposite goals such as capturing for pursuers and escaping for evaders.

Nowadays, the field of differential games has become an attractive research area with many applications in the field of economics, robotics and other many computer science related areas. Especially, the simple motion differential games with various types of geometric and integral constraints have already been analyzed in the

recent years and different types of improvements to the existing well-known strategies were introduced.

This paper will take a look on an improved strategy in the simple motion differential game with four pursuers and one evader where control functions of each player is subject to geometric constraints.

STATEMENT OF THE PROBLEM

Consider differential game with four pursuers and one evader described by the following

$$\begin{aligned} \dot{x}_i &= u_i, & x_i(0) &= x_{i0}, & |u_i| &\leq 1 \\ \dot{y} &= v, & y(0) &= y_0, & |v| &\leq 1 \end{aligned}$$

where $x_i, u_i, y, v, x_{i0}, y_0 \in \mathbb{R}^2, x_{i0} \neq y_0$, where $i \in \{1, 2, 3, \dots, m\}, m = 4$.

A function $G_i(x_i, y, v)$ is called the strategy of the pursuers x_i , if for any control of the evader $\dot{y} = v(t), t \geq 0$, the initial value problem

$$\begin{cases} \dot{x}_i = G_i(x_i, y, v), & x_i(0) = x_{i0} \\ \dot{y} = v, & y(0) = y_0 \end{cases}$$

where $G_i: \mathbb{R}^2 \times \mathbb{R}^2 \times H(0,1) \rightarrow H(0,1), H(0,1) = \{x \in \mathbb{R}^2 | |x| \leq 1\}$ has a unique solution $(x_i(t), y(t))$.

Construct improved strategy and algorithm for four pursuers and one evader

SUGGESTED IMPROVED STRATEGY

$$G_i(x_i, y, v) = \begin{cases} e_{(i,i+1)}, & \langle v, \mu_{(i,i+1)} \rangle > \langle e_{(i,i+1)}, \mu_{(i,i+1)} \rangle \\ e_{(i,i-1)}, & \langle v, \mu_{(i,i-1)} \rangle > \langle e_{(i,i-1)}, \mu_{(i,i-1)} \rangle \\ v - \langle v, e_i \rangle * e_i + e_i * \sqrt{1 - |v|^2 + \langle v, e_i \rangle^2}, & \text{otherwise} \end{cases}$$

VISUALIZATION OF NUMERICAL SOLUTION

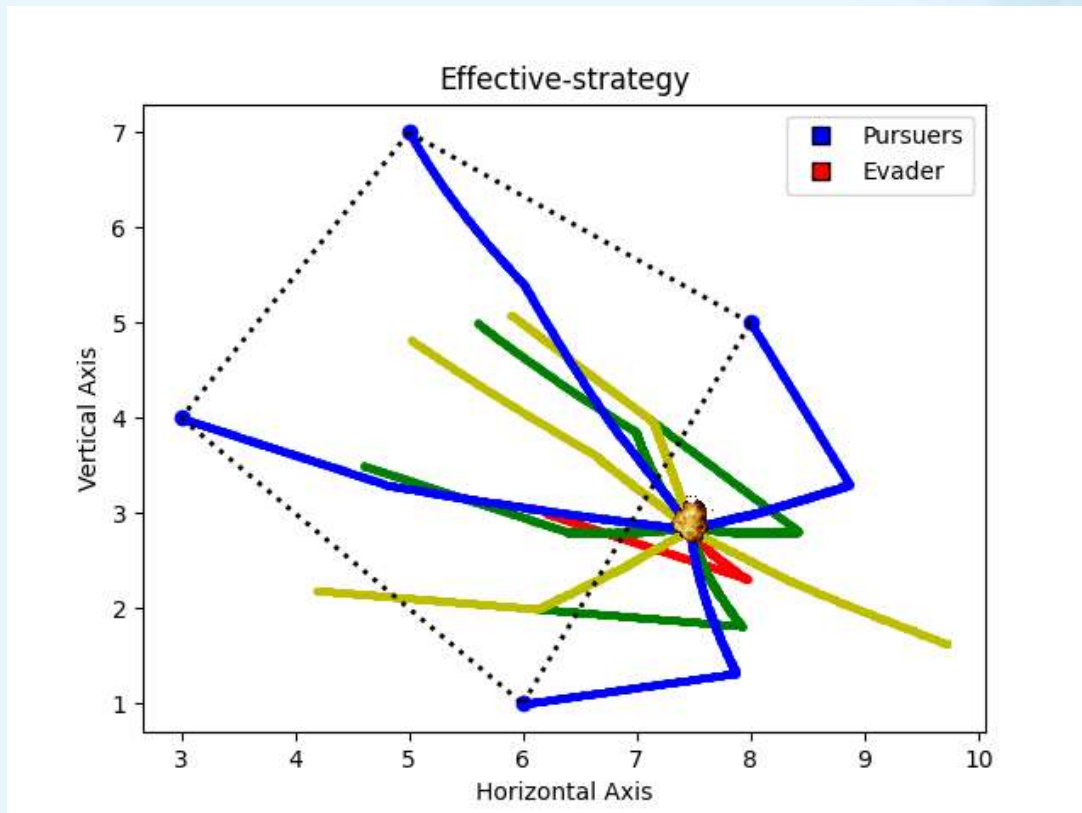


Fig. 1 $\chi_0 = \{(3,4), (5,7), (8,5), (6,1)\}$, $y_0 = (6.2,3)$

CONCLUSION

The zero-sum differential game with four pursuers and one evader was considered and studied. We have constructed an improved strategy and algorithm. Finally, we finished our work by doing simulations using Python to check our theoretical results.

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