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THREE-BODY PROBLEM: MATHEMATICAL APPROACH TO SIGNAL TRANSMISSION BETWEEN EARTH AND MOON VIA ARTIFICIAL SATELLITE

Safarmatov Uchqun Sohibjon o‘g‘li

Eshboyev Ilhom Ikrom o‘g‘li

Tashkent State Technical University Almalik Branch

Annotation: This paper presents a mathematical model of the process of information transmission between Earth, the Moon, and an artificial satellite. The three-body problem, which is of significant importance in space exploration and communication systems, examines the dynamics and mutual interaction of these bodies. The paper outlines the theoretical foundations of the equations of motion and orbital mechanics, as well as the methods for calculating the parameters of the signal (frequency, delay, interruption) transmitted via the artificial satellite. Numerical simulation methods are applied to model orbital positions and study the efficiency of signal transmission. These findings will contribute to the future development of telecommunication systems and enhance the effectiveness of space communications.

Keywords: Orbital motion; test simulations; space mechanics; doppler effect; wavelength; frequency; signal strength.

1. Introduction.

The gravitational interactions between the Earth, the Moon, and an artificial satellite, as well as their motion, are central to understanding the three-body problem. However, the challenge lies in the complexity of the three-body problem and the mathematical difficulties in finding an exact solution. The goal of this research is to develop a mathematical model for the process of data transmission between the Earth, the Moon, and an artificial satellite. This model will help analyze the efficiency of information transmission and the characteristics of the signal. To achieve this goal, the following tasks are necessary:

- Determining the equations of motion.
- Analyzing orbital mechanics.
- Modeling the data transmission process and presenting the results.

The three-body problem is considered one of the critical challenges in space exploration and telecommunications. The significance of this research lies in its application to optimizing the process of data transmission via an artificial satellite in space communication systems. Despite existing research, there are unresolved issues and challenges in this field.

2. Theoretical Foundations.

The three-body problem involves the dynamic motion between two primary bodies (the Earth and the Moon) and a third, smaller body such as an artificial satellite (used in space exploration). This problem is based on Newtonian mechanics, and there is no exact solution due to its complexity and inherent instability. To mathematically describe the three-body problem, we must present the equations of motion for each body. The equation of motion for each body is as follows:

$$\frac{d^2 r_i}{dt^2} = -G \sum_{j \neq i} \frac{M_j (r_i - r_j)}{|r_i - r_j|^3} \quad 1.$$

This equation defines the forces acting on each body and how these forces affect its motion. Here r_i *and* r_j represent the positions of the bodies, G- is the gravitational constant, and M_j- represents the mass of each body. These equations align with Kepler's laws of orbital motion

Kepler's laws:

1. **First law**: Each body moves in an elliptical orbit around a more massive body (e.g., the Earth or Moon).
2. **Second law**: A body's velocity increases as it approaches a massive body and decreases as it moves away.
3. **Third law**: There is a relationship between the orbital period and the semi-major axis of the orbit. For example, the orbital motion between the Earth and the Moon can be described by the following equation:

$$T^2 \sim a^3 \quad 2.$$

Where T - is the orbital period, and a - is the average radius of the orbit..

The interaction between the Earth, Moon, and the artificial satellite during orbital motion can be modeled using the following equations:

$$\frac{d^2x_E}{dt^2} = -\frac{GM_o(x_E - x_o)}{(x_E - x_o)^2 + (y_E - y_o)^{2^{3/2}}}$$

$$\frac{d^2y_E}{dt^2} = -\frac{GM_o(y_E - y_o)}{(x_E - x_o)^2 + (y_{E\_} - y_o)^{2^{3/2}}} \quad 3.$$

$$\frac{d^2x_o}{dt^2} = -\frac{GM_E(x_o - x_E)}{(x_o - x_E)^2 + (y_o - y_E)^{2^{3/2}}}$$

$$\frac{d^2y_o}{dt^2} = -\frac{GM_E(y_o - y_E)}{(x_o - x_E)^2 + (y_o - y_E)^{2^{3/2}}}$$

These equations must be solved numerically using methods such as Runge-Kutta or Euler's method to model the motion.

3. Experimental: Programming and Simulation

To solve the equations mentioned above, numerical methods such as the **Runge-Kutta method or Euler's method** are commonly used.

Euler's Method: This method is a straightforward numerical technique used to approximate the solution of first-order differential equations. However, it is not highly accurate and may lead to errors when the function's rate of change is steep. Euler's method is given as follows:

$$r_i(t + \Delta t) = r_i(t) + \mathrm{v}_i(t) \cdot \Delta t$$

4.

$$\mathrm{v}_i(t + \Delta t) = \mathrm{v}_i(t) + \frac{dr_i}{dt} \cdot \Delta t$$

This method lacks sufficient accuracy for higher-order equations, making it less reliable for modeling complex orbital dynamics..

Runge-Kutta Method:

The **Runge-Kutta method**, particularly the 4th-order version, is widely used for solving first-order differential equations with higher accuracy. It improves upon Euler's method by evaluating the function multiple times at each step, thus reducing error and providing more precise results.

The 4th-order Runge-Kutta method is represented by the following steps:

1. $k_1 = f(\mathrm{t}_n, \mathrm{y}_n)$
2. $k_2 = f(\mathrm{t}_n + \frac{h}{2}, \mathrm{y}_n + \frac{h}{2} k_1)$

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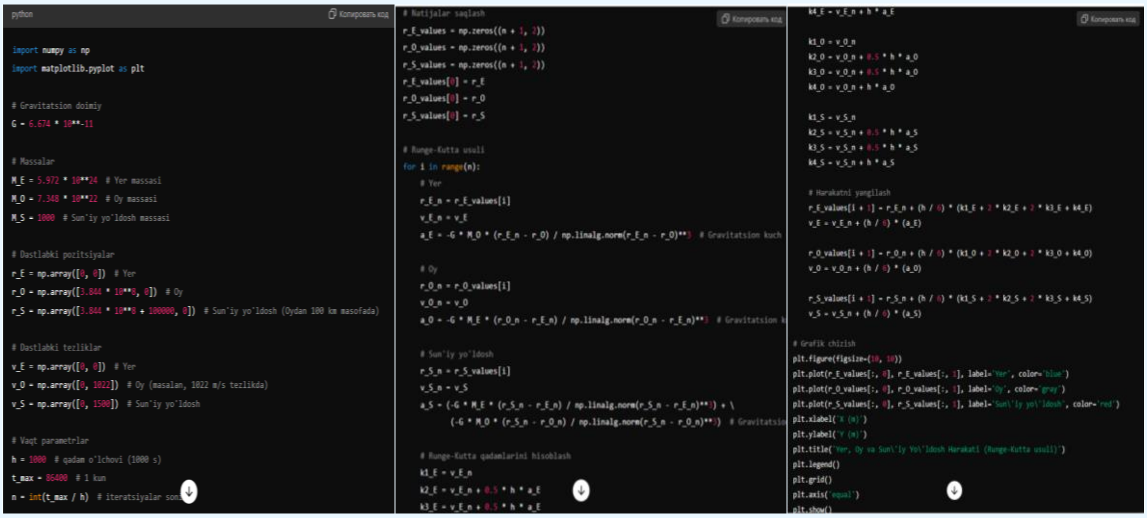
3. $k_3 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2)$

5.

4. $k_4 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_3)$

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Where$k_1, k_2, k_3, and\ k_4$ represent intermediate steps in calculating the future position and velocity of the objects, and hhh is the time step size. This method is highly accurate and commonly used for simulating orbital mechanics. Using the Runge-Kutta method, the motion of the Earth, Moon, and artificial satellite can be simulated. The numerical results can be visualized graphically by using programming languages such as Python.

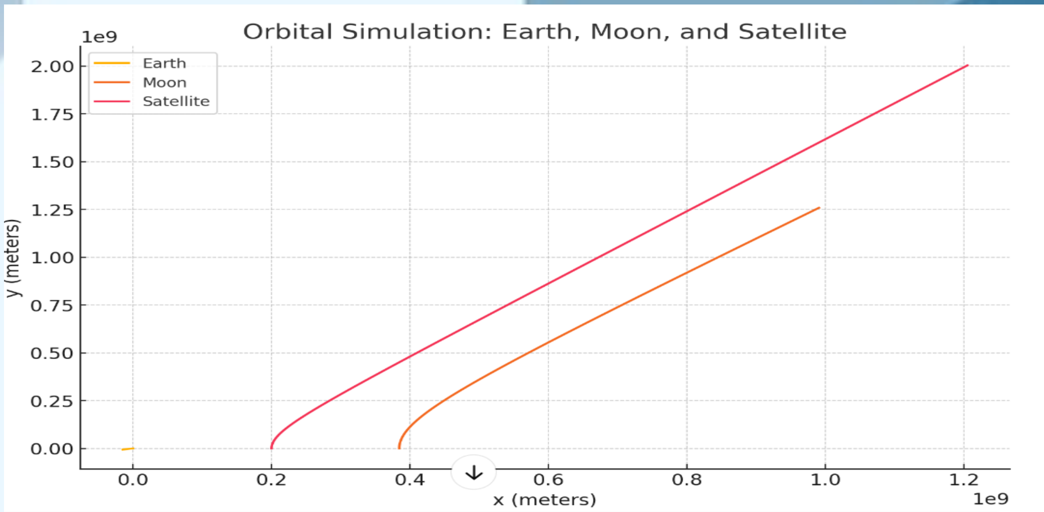


Visualization of the motion of the Earth, Moon, and artificial satellite in Python code.

Note:

1. A separate function was created to calculate gravitational forces.
2. Each position and velocity is updated.
3. The new positions of the Moon and the artificial satellite are updated at each step.
4. The graph is plotted using plt.show for visualization.

Difficulties and problems may arise during the modeling process, such as uncertainties in the dynamics of the forces.



Here is the graphical representation of the orbital simulation for the Earth, Moon, and satellite based on their gravitational interactions. The plot shows the trajectories of each object as they move under the influence of each other's gravity.

- The central point represents Earth.
- The orbit around Earth represents the Moon's trajectory.
- The satellite's path is shown as it interacts with both the Earth and the Moon.

4. *Data Transmission*

During data transmission via an artificial satellite, several key factors need to be considered:

Signal Strength: The relationship between transmitted and received signal strength can be described by the following formula:

$$P_r = \frac{P_t \cdot G_t \cdot G_r}{(4\pi R)^2} \qquad 6.$$

Where: P_r- is the received signal strength, P_t - is the transmitted signal strength, G_t va G_r - are the gains of the transmitting and receiving antennas, R is the distance between the satellite and the receiving station. This formula shows that as the distance R increases, the received signal strength P_r decreases proportionally to the square of the distance. For example, if the satellite is far from the receiving station on Earth, the signal strength decreases significantly.

Doppler Effect: The **Doppler Effect** occurs when the frequency of a signal changes due to the relative motion of the transmitter or receiver. When the satellite moves between the Earth and the Moon, its velocity affects the received signal frequency. This change in frequency is described by:

$$f' = f(\frac{c+v_r}{c})$$

7.

f'- is the received frequency, f- is the transmitted frequency, c is the speed of light, v_r is the relative velocity of the satellite. If the satellite is moving toward Earth, the received frequency increases; if it is moving away, the frequency decreases. This effect is particularly important in high-frequency communication systems, where even small velocity changes can significantly affect signal reception.

Xulosa: In this article, a mathematical model for signal transmission between the Earth and the Moon via an artificial satellite was analyzed. The three-body problem's orbital dynamics, signal strength, and frequency shifts were evaluated both theoretically and using numerical methods. The presented model offers insights into future space communication technologies, highlighting important technological approaches. As the satellite moves, the signal frequency shifts due to the Doppler Effect. As the relative velocity increases, the received frequency also rises. Additionally, the signal strength received from the satellite changes inversely with the square of the distance between the satellite and the receiving station. The greater the distance, the weaker the signal becomes.

References:

1. **Keller, J.B. & Kofman, M.** (2004). *Mathematical Methods for Physics and Engineering*. Cambridge University Press.
2. **Hale, J.K. & Kocak, H.** (2006). *Dynamics and Bifurcations of Nonlinear Systems*. Springer.
3. **Kleiber, M. & Becher, M.** (2010). *Mathematical Modeling of Satellite Orbits: A Practical Approach*. Springer.
4. **Safarmatov Uchqun Sohibjon o‘g‘li & Abdullayev Botir Ismoilovich** (2024). *The Possibilities and Importance of Heat and Electrical Conductivity of Modern Materials in Production*. Retrieved from https://confrencea.one/index.php/25-27/article/view/35/24

5. **Kuchkarov Shohjahon Xazrat o‘g‘li & Irkinov Ma’murjon** (2024). *Physical Properties of Composite Materials and Their Importance in Production*. Retrieved from https://confrencea.one/index.php/25-27/article/view/36/25

6. **Korshunova, N.A. & Eshboyev, I.I.** (2021). *Stabilizing the Motion of Intermediate Gravitational Systems for the Restricted Three-Body Problem*. International Scientific and Practical Conference "Innovation-2021". Tashkent: Innovative Development Publishing House.

7. **Korshunova, N.A. & Eshboyev, I.I.** (2021). *Stabilizing the Motion of Intermediate Gravitational Systems for the Restricted Three-Body Problem*. Republican Scientific Conference on "Mathematics, Mechanics, and Intellectual Technologies". Uzbekistan Mathematics Society, Institute of Mathematics, V.I. Romanovskiy, Uzbekistan Academy of Sciences.