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ELEMENTS OF GALILEY GEOMETRY

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Annotation:

In this article, with the helpof the usage of the elements of Galilean geometry, a number of properties are given for shapeson the ground, ie triangles. It is noted that the formulas are similar to the formulas in Euclidean geometry. New formulas similar to Euclidean geometry were found, given the Galilean meanings of some concepts. Although the concepts in Euclidean geometry are used in the concepts in this article, their geometric meanings are radically different. The properties created in the article will greatly help to solve new problems in planimetry and also to understand Galilean geometry.

Key words: triangle, triangle's surface, angle, height, parabola, cycle, Galilean geometry.

Galilean geometry is the part of Pseudo-Euclidean geometries. Pseudo-Euclidean geometry differs from Euclidean geometry in scolar product [4]. First of all, let's get acquainted with the basic concepts of Galilean geometry.

There is give A_2 in affin space $\vec{X}(x_1, y_1)$ and $\vec{Y}(x_2, y_2)$ vectors.

Description: $\vec{X}(x_1, y_1)$ and $\overrightarrow{Y}(x_2, y_2)$ scolar product both vectors

$$\begin{cases} (XY)_1 = x_1x_2\,; \\ \quad (XY)_2 = y_1y_2, \quad if\ (XY)_1 = 0 \end{cases}$$

found A_2 in the affin space R_2^1 is called Galileon gemetry. [3]

Average amount of vector in R_2^1 space is equal to the root of the scolar product of vector itself

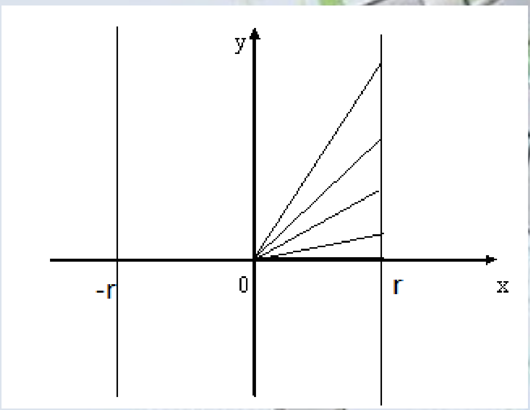
$$\|\vec{X}\| = \begin{cases} x_1 & when \quad x_1 \neq 0 \\ y_1 & when \quad x_1 = 0 \end{cases}.$$

R_2^1 space $A(x_1, y_1)$ and $B(x_2, y_2)$ the norm of vector joined both dots is called the distance between two dots.[1]

$$|\overrightarrow{AB}| = \begin{cases} |x_2 - x_1| & if \quad x_2 \neq x_1 \\ |y_2 - y_1| & if \quad x_2 = x_1 \end{cases}$$

This distance in the Galilean plane is its meaning in the Euclidean plane. The projection of the cross section connecting the two points on the OX axis.If the projection on the OX axis is a point, then the projection on the OY axis is obtained.The geometric position of points equidistant from a given point is called a circle. In the Galilean plane, the circle looks like this (Figure 1). [1]

$$x^2 = r^2; x = \pm r.$$



1-figure

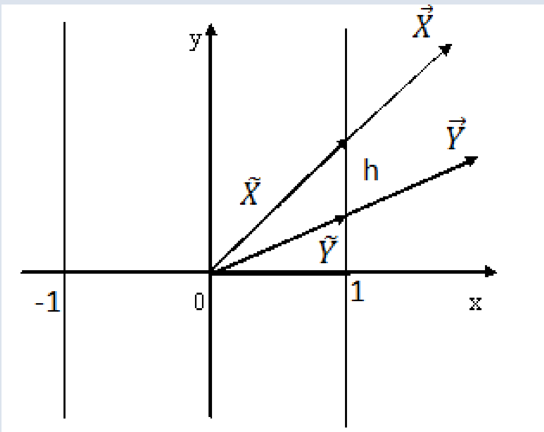
If in R_2^1 space $\vec{X}(x_1, y_1)$ and $\vec{Y}(x_2, y_2)$ vectors are given, the unity of this vectors will be like:.

$$\tilde{X}\left(1, \frac{y_1}{x_1}\right), \tilde{Y}\left(1, \frac{y_2}{x_2}\right);$$

The angle between $\vec{X}$ va $\vec{Y}$ vectors is calculated as following:

$$h = \left|\frac{y_2}{x_2} - \frac{y_1}{x_1}\right|. \qquad (1).$$

The geometric meaning of the angle formed in the Galilean plane is to be understood as follows. We place the heads of the vectors at one point, then draw the unit circle of the Galilean plane with the heads of the vectors at the center. and the cross-sectional length formed between the vectors is the angle between these two vectors (Figure 2).



2-figure

Lines parallel to the axis in the Galilean plane are called special lines.

Using the concepts of Galilean geometry in the plane, a number of properties and formulas for geometric shapes such as straight lines, triangles, circles, parabolas are given in [1]. In the Galilean plane, some properties of second-order lines, such as ellipses, hyperbolas, and parabolas, are shown in [7]. In three-dimensional space, the Galilean substitution for the second-order sursurface has excellent properties [8]. In contrast, we show several theorems using the concepts of Galilean geometry for the surface of a triangle in a plane.

We are going to find the surfaces of polygons whose arbitrary sides do not lie on a special line.

1.If there is given triangles that angles are on $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ dots . We take BC as a basis and AH as height (3-figure), the surface of triangle is equal to: [6]

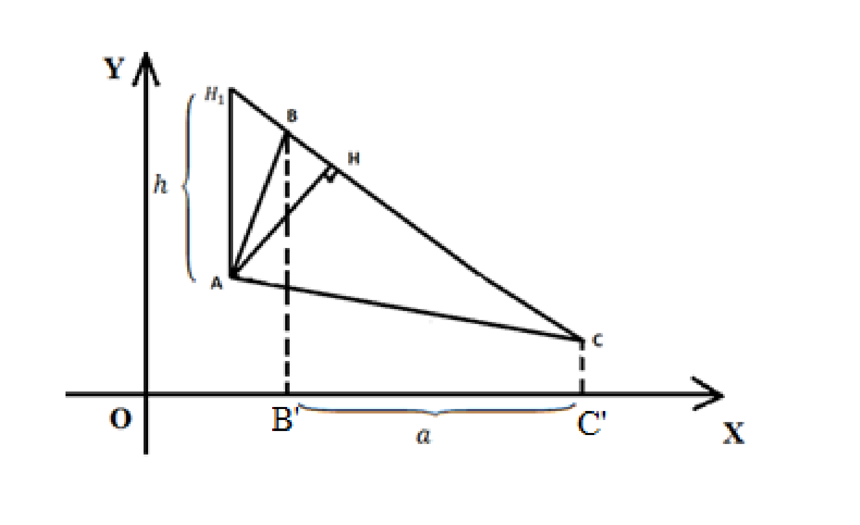
$$S = \frac{1}{2}\begin{vmatrix} x_2 - x_1 & y_1 - y_2 \\ x_3 - x_2 & y_2 - y_3 \end{vmatrix}$$

We calculate the surface of a triangle using elements of Galilean geometry.

We get the length of the side of the triangle in the Galilean sense, ie the cross section B`A`. By taking the height parallel to the axis of OY from the opposite side of the side, we get the section drawn in this direction, that is, the section AH`

Theorem 1: The surface of a triangle in a plane is equal to:

$$S = \frac{1}{2}ah' = \frac{1}{2}|B'C'| \cdot |AH'|$$



3-figure

Proof: There is given ABC triangle. We can say for clarity $x_1 < x_2 < x_3$ For the basis of triangle we can take the side $B'C' = a = x_3 - x_2$ and we find the height belongs to that. All things considered, The equation of special straight line passing through A $x = x_1$ as well as straight line passing through B and C is like this: $\frac{x-x_2}{x_3-x_2} = \frac{y-y_2}{y_3-y_2}$

To find these both straight line:

$$x = x_1, y = \frac{x_1-x_2}{x_3-x_2}(y_3 - y_2) + y_2$$

Because of being of the heighton the special line, its length is equal to:

$$h' = \frac{x_1-x_2}{x_3-x_2}(y_3 - y_2) + y_2 - y_1 = \frac{\begin{Vmatrix} x_2-x_1 & y_2-y_1 \\ x_3-x_2 & y_3-y_2 \end{Vmatrix}}{x_3-x_2}$$

If we consider that the surface of a triangle is equal to one-half of the product of the height of the base and the surface of the triangle is equal to

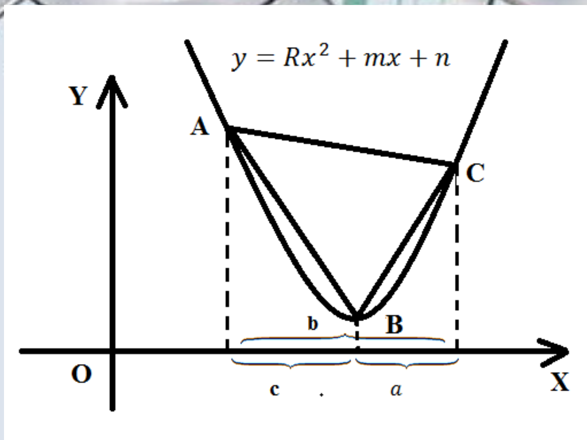
$$S = \frac{1}{2}a'h' = \frac{1}{2}\begin{Vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_2 & y_3 - y_2 \end{Vmatrix}$$

Theorem is proved.

This means that the sursurface of a triangle can also be calculated using the concepts of length and height in helium geometry. This helps us to easily solve new problems. Most importantly, in Euclidean geometry and Galilean geometry, the concepts are different, but the formulas are semantically similar and the numerical values of the sursurface are the same.

Even if we take the other side as the basis and take the height in the Galilean sense relative to that side, the sursurface of the triangle will look similar.

2.If there is given triangle angles on the dots $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ dots, triangle sides are equal to: $a = |x_3 - x_2|$, $b = |x_3 - x_1|$, $c = |x_2 - x_1|$ and there is given parabola passing through these dots $y = Rx^2 + mx + n$ (4-figure).



4-figure

Theorem 2: the surface of ABC triangle is equal to:

$$S = \frac{1}{2} Rabc$$

Proof: Because of the parabola $y = Rx^2 + mx + n$ passed through A, B and C dotsn we will organize the system as following:

$$\begin{cases} y_1 = Rx_1^2 + mx_1 + n \\ y_2 = Rx_2^2 + mx_2 + n \\ y_3 = Rx_3^2 + mx_3 + n \end{cases}$$

Subtracting the first line from the second line of this system, the second line from the third line, we come to the following system.$\begin{cases} y_2 - y_1 = R(x_2^2 - x_1^2) + m(x_2 - x_1) \\ y_3 - y_2 = R(x_3^2 - x_2^2) + m(x_3 - x_2) \end{cases}$

After multiplying the first line of this system to $x_3 - x_2$, the second one to $x_2 - x_1$ we wil subtract then the following equotion will appear.

$$(y_2 - y_1)(x_3 - x_2) - (y_3 - y_2)(x_2 - x_1) = R[(x_2^2 - x_1^2)(x_3 - x_2) - (x_3^2 - x_2^2)(x_2 - x_1)]$$

If we simplify the equation like this:

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_2 & y_3 - y_2 \end{vmatrix} = R(x_2 - x_1)(x_3 - x_2)(x_3 - x_1).$$

Because of this:

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_2 & y_3 - y_2 \end{vmatrix} = S$$

The following equation will appear:

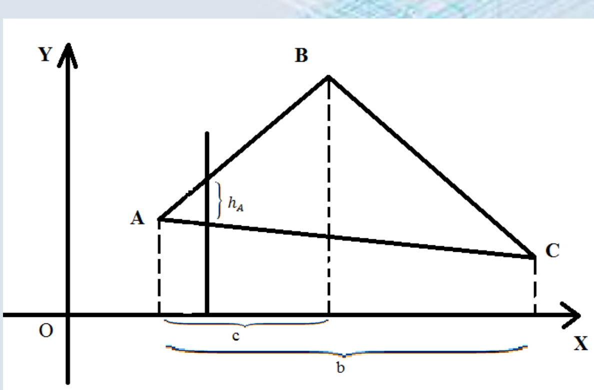
$$S = \frac{1}{2} Rabc.$$

Theorem is proved.

Explanation: parabola $y = Rx^2 + mx + n$ is called a cycle in the Galilean plane, it has many properties of a circle in the Euclidean plane, i.e. a circle in the Galilean plane. The resulting formula has the same meaning as the formula for the radius of a circle drawn inside a triangle in Euclidean geometry.

Note: For an inscribed triangle with sides k, l, m on a circle of radius r in Euclidean geometry $S = \frac{1}{2} R(a + b + c)$[8]

If there is given triangle which its angles on the dots $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ we will draw a special straight line at a unit distance from one tip of this triangle we may call the angle of the triangle the length of the intersection of this straight line formed by the intersection of the sides of the triangle the angle of the triangle [1] (Fig. 5).



5-figure

The length of AB side of triangle $c = |x_2 - x_1|$, the length of the side AC is equal to $b = |x_3 - x_1|$ and the angle of tip A is equal to h_A.

Theorem 3: The surface of ABC triangle is equal to:

$$S = \frac{1}{2} bch_A$$

Proof: The way of finding the angle h_A between the sides b and c (1) is:

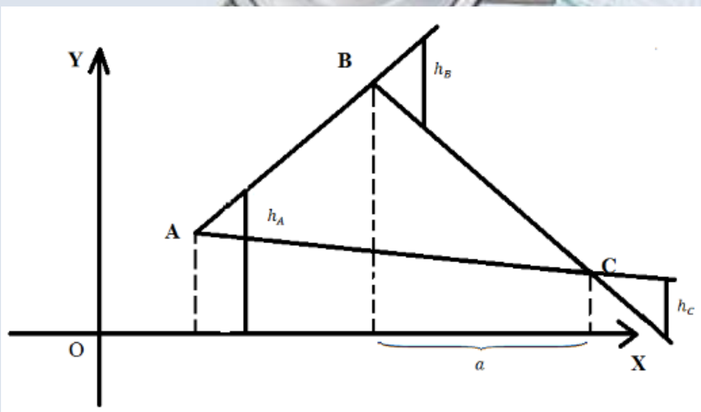
$$h_A = \left| \frac{y_2 - y_1}{x_2 - x_1} - \frac{y_3 - y_1}{x_3 - x_1} \right| = \frac{\left\| \begin{matrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_2 & y_3 - y_2 \end{matrix} \right\|}{|x_2 - x_1||x_3 - x_1|} \quad (*)$$

With help of theorem 1 we can achieve these results:

$$S=\frac{1}{2}ah'=\frac{1}{2}\left\|\begin{matrix}x_2-x_1 & y_2-y_1\\ x_3-x_2 & y_3-y_2\end{matrix}\right\|=\frac{1}{2}|x_2-x_1||x_3-x_1|\frac{\left\|\begin{matrix}x_2-x_1 & y_2-y_1\\ x_3-x_2 & y_3-y_2\end{matrix}\right\|}{|x_2-x_1||x_3-x_1|}$$
$$=\frac{1}{2}bch_A$$

So $S=\frac{1}{2}bch_A$. Theorem is proved.

There is given us the triangle tips on the dots $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ the angle of tip A is equal to h_A, the angle of tip B is equal to h_B, the angle of tip C is equal to h_C and the side which is opposite angle A is equal to $a=|x_3-x_2|$ (6-figure).



6-figure

Theorem 4: The surface of triangle ABC is equal to

$$S=\frac{a^2h_Bh_C}{2h_A}$$

Proof: Accordig to (1) the angles of triangle is equal to $h_B=\left|\frac{y_1-y_2}{x_1-x_2}-\frac{y_3-y_2}{x_3-x_2}\right|$, $h_C=\left|\frac{y_1-y_3}{x_1-x_3}-\frac{y_2-y_3}{x_2-x_3}\right|$ va $h_A=\left|\frac{y_2-y_1}{x_2-x_1}-\frac{y_3-y_1}{x_3-x_1}\right|$ Each of these angles is made like (*) and with the help of the surface of triangle at the first theorem we can create followings:

$$S=\frac{1}{2}\left\|\begin{matrix}x_2-x_1 & y_2-y_1\\ x_3-x_2 & y_3-y_2\end{matrix}\right\|$$

It is derived from the formula for the surface of a triangle found above. The theorem is proved.

During the study of the above theorems, students will be able to find the surface of a triangle and get an idea of non-Euclidean geometry.

In conclusion, we can talk about non-Euclidean geometry. Any geometry that is different from Euclidean geometry is called non-Euclidean geometry. Examples include Lobachevsky geometry, Riemannian geometry, Galilean geometry, and others. In total, there are 9 geometries in the plane and 27 in space. [9,10]

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