MODEL OF PROPAGATION OF RADIO WAVES IN FRACTAL MEDIUM

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Introduction. It is known that when a wave propagates through a turbulent medium, the amplitude and phase of this wave will generally fluctuate depending on the fluctuations in the refractive index of the medium. In optics, radiophysics, and acoustics, the classical statistical theory has been scientifically proven in detail when considering waves [1-7]. But the issues of wave propagation in fractal media, which are important in modern independent scientific direction, are not systematized and not widely covered in scientific literature.

The problem of investigating the propagation of waves through fractal media is mainly determined by various approximations [1-7]. It is difficult to verify the appropriateness of such approaches due to the neglect of some externalities. It seems natural to look for such a problem in wave propagation that gives accurate results used as a test for standard approximations. A propagation medium whose length is L with dielectric permittivity $\varepsilon(x)$ characterized by the Cantor model [7-11] is considered. In the $n-$ step, one third of the interval ε_n is subtracted from the value generated by $\varepsilon(x)=\varepsilon_0$, and ε_n in the remaining intervals is increased by 1.5 times.

$\varepsilon(x)$directly ε_0 is obtained by an infinitely large number of such iterations of finite dimension. Let a harmonic plane wave $\exp(ikx)$ be incident on this layer from the right.

The wave function $U(x)$ satisfies the Helmholtz equation in the interval $[0,\ L_0]$ [7-8]:

$$\frac{d^2U(x)}{dx^2}+k^2(1+\varepsilon(x))U(x)=0 \tag{1}$$

where: $k-$wavenumber.

Outside the environment, when $x > L_0$ [7-8]:

$$U(x) = \exp(ikx) + V_0 \exp(ikx) \tag{2}$$

and when $x < 0$ [7-8]:

$$U(x) = T_0 \exp(-ikx) \tag{3}$$

where, V_0 and T_0 – are the complex reflection and transmission coefficients of the layer respectively.

$|T_0|^2 = 1 - |V_0|^2$ value is found that describes the fraction of energy passing through the fractal medium. For this, the scattering matrix formalism is used.

Let $L_n = \frac{L_0}{3^n}$ be the scattering matrix $\hat{S}$ – connecting the right and left amplitudes of a wave incident on a fractal medium of duration.

A contour fractal consists of two identical layers separated by an empty space, then can be written as [2-7]:

$$\hat{S}_n = \hat{S}_{n+1} \hat{F}_{n+1} \hat{S}'_{n+1,} \tag{4}$$

where, $\hat{F}_n \quad diag\left\{\exp\left(-ikL_n,\ \exp\left(ikL_n\right)\right)\right\}$ – is the diagonal matrix of transmission in free space.

In the proposed system of difference equations, where $\hat{S}'_n$ and $\hat{S}'_{n+1}$ are represented by the corresponding complex coefficients T_m and $V_m (m = n,\ n+1)$, from equation (4) [7-13]:

$$\begin{cases} \frac{1}{|T_n|^2} - 1 = \frac{4\cos^2(\varphi_{n+1} + kL_{n+1})}{|T_{n+1}|^2}\left(\frac{1}{|T_{n+1}|^2} - 1\right), \\ tg\varphi_n = \frac{\sin(2\varphi_{n+1} + kL_{n+1}) - (1 - |T_{n+1}|^2)\sin(kL_{n+1})}{\cos(2\varphi_{n+1} + kL_{n+1}) + (1 - |T_{n+1}|^2)\cos(kL_{n+1})}, \end{cases} \tag{5}$$

To solve the equations (5), they are filled with conditions arising from T_n asymptotics when $n \to \infty$. It should be noted that large values of n correspond to passing through a fractal of thickness $L \to \infty$. Under this assumption, the equation (1) gives the continuity equation for the wave $U_n(x)$ propagating in the fractal as follows [7-13]:

$$U_n(x)=T_n\exp(-ikx)+k\int_0^x dx'\sin[x'-x]\varepsilon_n(x')U_n(x'). \tag{6}$$

The transfer coefficient T_n is determined in terms of $x=L_n$. In the limit $L_n\to 0$, $U_n(L_n)$ quantity is approximated by the following expression [7-13]:

$$U_n(L_n)\approx T_n\exp(-ikL_n)-\sin(kL_n)U_n(x'=0)\frac{\varepsilon_0 kL_0}{2^n} \tag{7}$$

Combining the expression (7) for U_n' with the corresponding expression for $n\to\infty$, the following can be obtained for $x=L_n$ [8-15]:

$$T_n\infty\left(1-\frac{ik\varepsilon_0 L_0}{2^{n+1}}\right)^{-1} \tag{8}$$

The system of equations (5) gives the necessary solution of the problem with the expression (8). The maximum number of steps N in the numerical solution of the expression (5) is found with the accuracy of the solution. Considering [2-8], the value of N is selected in the range of $24\div$ 35. s. The method of phenomenological modeling of the local homogeneous refractive index of the troposphere based on the fractal concept was considered by Y. Kim and D. L. Jaggard [7]. Weierstrass band-limiting functions, which are widely used in the study of fractals, were used in this [2-9]. It is often convenient to express the spectrum of turbulence in several time intervals and consider their basis functions. [8-10] of the turbulence spectrum is modeled on a logarithmic scale at the terminals of sequences of self-similar pulses when the Fourier localizes the medium. The scales are in the interval from the inner l scale to the outer L scale, so that b must satisfy the ratio $b^N=\frac{L}{l}$ with the fundamental spatial frequency or the spatial-frequency scaling parameter. The situation in which the mobility characteristics of the Weierstrass function are negated [3-4] it will be necessary to consider. This means that $b \to 1$ and all scales are spatially filled due to the absence of a localization function. Then the correlation function $R(x')$ averaged over φ_1 is [9-15]:

$$R(x') = \frac{p_1^2 \langle n_f^2 \rangle \left[1 - b^{2(D-2)}\right]}{1 - b^{2(D-2)(N+1)}} \sum_{n=0}^{N} b^{2(D-2)n} \cos\left(\frac{2\pi b^n x'}{L}\right). \tag{9}$$

The one-dimensional energy spectrum $L(k)$ corresponding to the expression (11) is defined as follows [8]:

$$L(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R(x') \exp(ikx') dx' \tag{10}$$

Substituting expression (9) into expression (10) and considering only positive spatial frequencies, we get [8]:

$$L(k) = \frac{p_1^2 \langle n_f^2 \rangle \left[1 - b^{2(D-2)}\right]}{2\left[1 - b^{2(D-2)(N+1)}\right]} \sum_{n=0}^{N} b^{2(D-2)n} \sigma\left(k - \frac{2\pi b^n}{L}\right), \tag{11}$$

Since $b \to 1$, the discrete wavenumbers in the expression (13) are almost continuous.

If we put $b = 1 + X$, the expression (11) can be rewritten in another form, where $X \to 1$ and $b^{(N+1)} \approx b^N = \frac{L}{l}$ [8]:

$$L(k) = \frac{p_1^2 \langle n_f^2 \rangle (4 - 2D) X}{2\left[1 - \left(\frac{L}{l}\right)^{2(D-2)}\right]} \sum_{n=0}^{N} b^{2(D-2)n} \sigma\left(k - \frac{2\pi b^n}{L}\right), \tag{12}$$

When $D = \frac{5}{3}$ and $L \gg 1$, the expression (12) becomes [8]:

$$L(k)=\frac{p_1^2\langle n_f^2\rangle X}{3}\sum_{n=0}^{N} b^{-\frac{2n}{3}}\sigma\left(k-\frac{2\pi b^n}{L}\right). \qquad (13)$$

Therefore, the corresponding one-dimensional spectrum is[8-11]:

$$L(k)=P_1^2\langle n_f^2\rangle\left(1-b^{-\frac{2}{3}}\right)\sum_{n=0}^{N} b^{-\left(\frac{2}{3}+\alpha\right)n}\frac{d\sin^2\left[db^{-\alpha n}\frac{\left(k-\frac{2\pi b^n}{L}\right)}{2}\right]}{4\pi\left[db^{-\alpha n}\left(\frac{\left(k-\frac{2\pi b^n}{L}\right)}{2}\right)\right]^2} \qquad (14)$$

It should be noted that the expression (14) approximates the expression (13) when $\alpha\to 0$ and $d\to\infty$. $0<\alpha\le 2/3$ is obtained from the expression. The value of b is determined from the condition $b-1<<L/d$, because with the increase of the value of d, smaller details begin to appear in the spectrum. L spectra with different α values are indistinguishable from the $k^{-\frac{5}{3}}$ curve. This means that [7-15] the model is insensitive to the values of α and b in the interval $b-1<<L/d$, therefore, together with the expression [8], this condition confirms the hypothesis that the spectrum of the Weierstrass function limited in range is a good approximation of the spectrum of A.N. Kolmogorov.

CONCLUSIONS

1. In the study of random signals for the modeling of disturbances and noise, they are approximated by modifications of the Weierstrass-Mandelbrot fractal function, it was found that the most appropriate approximation function for modeling the fluctuations of the refractive index of radio wave propagation in a turbulent tropospheric environment based on the fractal approach is the Weierstrass function, which is widely used in fractal research.

2. Based on the propagation of flat electromagnetic waves in an environment with dielectric absorption with weak fluctuations, an improved fractal model and calculation algorithm of the dielectric absorption of the troposphere depending on two variables for the state of fractal dimension 2<D<3 was developed.

3. The model of phase effects of dielectric absorption fluctuations, noise (noise) and their functional dependence on plane electromagnetic wave propagating in the troposphere layer has been developed.

4. An information-graphic model of the flat electromagnetic wave propagation process in the fractal troposphere and an expression of the interdependence of its components, a flat wave incident in the fractal tropospheric environment, dielectric absorption fluctuation and transfer functions of the generator and a calculation algorithm were created.

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